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Calculation of Field Manning's Roughness Coefficient

Zhe Li*, Juntao Zhang

Institute of Geography, Chinese Academy of Sciences, Beijing, PR China

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Abstract

Manning's roughness coefficient is one of the most important parameters for describing water flow over the ground. However, it is not well established how to determine this roughness parameter in a particular field situation of surface irrigation. In this paper, a method for calculating Manning's roughness coefficient is presented, based on an analytical model for border irrigation and solved by iteration. The results are satisfied via testing with the field data obtained from different regions and various soil types and situation of field surface. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Manning's roughness coefficient; Border irrigation; Border irrigation model

1. Introduction

Selection of a specific surface roughness value in a particular field situation of surface irrigation is not well established. It is usually estimated according to the actual situation in most cases. Roughness in surface irrigation has been expressed often in terms of Manning's roughness coefficient, *n*. Some present methods for estimating Manning's roughness (Harun-ur-Rashid, 1990) require one to collect data of the surface water depth in the water advance phase. However, it is difficult to obtain them in the field experiments. Therefore, they may be not accurate for application. This study was undertaken to develop a technique for calculating Manning's roughness coefficient *n*, in which the data of surface water depth are not required.

* Corresponding author. Tel.: +86-10-64856497.
E-mail address: lizhe917@sina.com (Z. Li).

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2. Calculation

2.1. Development of model

Irrigation water flow is the inconstant flow in open channel. The analytical model for border irrigation (YSM) (Yu and Singh, 1989) is a simple analytical model, which uses the volume balance approach to simulate all phases of border irrigation. A comparison with some existing models shows that the model is simpler, more accurate and easier to apply. In this paper, YSM is simplified and a new model developed to estimate Manning's roughness coefficient n.

In YSM, the following assumptions are made: (1) the inflow is constant; (2) the border is homogeneous (i.e. its slope, width, and composite roughness do not vary with location); (3) the border is sufficiently wide so that the side effects can be neglected; and (4) bed slope is so small that $\sin\alpha \cong \tan\alpha \cong S_0$ and $\cos\alpha \cong 1$, where α is the angle that the border makes with the horizontal plane. Additionally, the surface and subsurface flow profiles in the advance phase are assumed to be of parabolic shape, and their coefficients are determined by conditions in the gradually varied flow region, rather than in the rapidly varied flow region near the advance front. Meanwhile, it is assumed that the flow in the region from the upstream end to the section that is 2 m behind the front is gradually varied.

For an irrigating border, let a unit width constant inflow be q_0 (m^3/min/m), introduced at the head. Let the advance front position be denoted as S (m) for a given time t (min). The integral form of continuity equation can be expressed as

$$q_0 t = \int_0^S h(x,t)\,\mathrm{d}x + \int_0^S H(x,t)\,\mathrm{d}x \tag{1}$$

where x (m) is the distance along the soil surface from the upstream end ($0 \le x \le S$); $h(x, t)$ and $H(x, t)$ the surface and subsurface water profiles, respectively. The surface water depth $h(x, t)$ is assumed to be of parabolic form

$$h(x,t) = h_0(a_1 x^2 + b_1 x + c_1) \tag{2}$$

where a_1, b_1, c_1, are three coefficients that are functions of time only; and h_0 (m) the normal inflow depth and can be computed by Manning's Eq. (3), and the upstream boundary conditions may be written as (4) and (5).

$$h_0 = \left(\frac{n q_0}{60 S_0^{1/2}}\right)^{3/5} \tag{3}$$

$$h(0,t) = h_0, \quad 0 \le t < T_\mathrm{s} \tag{4}$$

$$\frac{\partial h(0,t)}{\partial x} = 0, \quad 0 \le t < T_\mathrm{s} \tag{5}$$

where n is Manning's roughness coefficient; S_0 (dimensionless) the bed slope; T_s the total inflow time. Although Eq. (5) is an approximation, it would deviate little from reality because the surface flow rate is much faster than infiltration rate. Hence, the opportunity times in the neighborhood of the upstream end are nearly equal. For many practical border slopes, the normal flow depth can soon be maintained in the neighborhood of the upstream end. Because the flow near the advance front is rapidly varied, the flow condition at the front cannot reflect the gradually varied flow profile behind the front. It may be preferable to find the third condition to determine the three coefficients in Eq. (2) in the gradually varied flow region. As the last assumption mentioned above in YSM, Kostiakov infiltration equation is employed as

$$H = Kt^{\alpha} + ct \tag{6}$$

where H (m) is the infiltrated water depth, and K (m/min$^{\alpha}$), α (dimensionless), and c (m/min) the three constant coefficients that can be determined by field experiments. As a matter of fact, only two coefficients suffice, thus, reducing work in the field. So, it may be expressed as

$$H = Kt^{\alpha} \tag{7}$$

For uniform advance, the opportunity time along the border is linear so that the average infiltration rate for an advance distance S and time t can be calculated by using Eq. (7) as

$$I = \frac{1}{t}\int_0^t K\alpha t^{\alpha-1}\,\mathrm{d}t = Kt^{\alpha-1} \tag{8}$$

For gradually varied flow, Eq. (8) may be employed by multiplying a nonuniform flow coefficient f (dimensionless) so that the flow rate at the section $x = S - 2$, q_2 (m^3/min/m) may be estimated by neglecting the rate of change of surface storage in the range of $0 \le x \le S - 2$.

$$q_2 = q_0 - fI(S-2) \tag{9}$$

where the nonuniform flow coefficient f was calibrated using one data set from Ley's thesis (Ley, 1978). The value of $f = 0.35$ gave the least sum of the squared errors between observed and computed values. A sensitivity of analysis of this parameter, based on Lay's data set (Ley, 1978), showed that the YSM advance function was relatively insensitive to the parameter f. When f was changed from 0.25 to 0.40, the calibration error changed within 1%.

On the other hand, q_2 can be computed by Manning's equation as

$$q_2 = \frac{60S_0^{1/2}h_2^{5/3}}{n} \tag{10}$$

Equating Eqs. (9) and (10) yields

$$h_2 = \left\{[q_0 - fKt^{\alpha-1}(S-2)]\frac{n}{60S_0^{1/2}}\right\}^{3/5} \tag{11}$$

Therefore, the third boundary condition can be given as

$$h(S-2, t) = h_2 \tag{12}$$

Application of Eqs. (4), (5) and (12) to Eq. (2) produces

$$a_1 = 1 \tag{13a}$$

$$b_1 = 0 \tag{13b}$$

$$c_1 = \frac{-(h_0 - h_2)}{h_0(S-2)^2} \tag{13c}$$

Hence, the surface water profile becomes

$$h(x,t) = h_0 - \frac{(h_0 - h_2)x^2}{(S-2)^2} \tag{14}$$

Then the volume of water remaining on the surface, V_{sa} (m^3), for an advance distance S and time t is

$$V_{sa} = \int_0^S h(x,t)\,dx = Sh_0 - \frac{(h_0 - h_2)S^3}{3(S-2)^2} \tag{15}$$

Similarly, the subsurface water profile is also assumed to be of parabolic form

$$H(x,t) = H_0(a_2x^2 + b_2x + c_2) \tag{16}$$

where H_0 (m) is the infiltrated water depth at the upstream end for a given time t and can be computed by Eq. (7), and a_2, b_2, and c_2 the three coefficients that are functions of time only. The upstream end boundary conditions may be specified as

$$H(0,t) = H_0 \tag{17}$$

$$\frac{\partial H(0,t)}{\partial x} = 0 \tag{18}$$

Since the flow at the section that is 2 m behind the front was assumed to be gradually varied in YSM, the mean velocity, v_2, at this section may be computed by Manning's equation as

$$v_2 = \frac{q_2}{h_2} = \frac{60S_0^{1/2}h_2^{2/3}}{n} \tag{19}$$

The opportunity time, t_2, at the section $x = S-2$ may be estimated by assuming that the mean velocity from $x = S-2$ to $x = S$ is one half of the flow velocity at section $x = S-2$

$$t_2 = \frac{2}{0.5v_2} = \frac{n}{15S_0^{1/2}h_2^{2/3}} \tag{20}$$

Therefore, the infiltrated water depth at section $x = S-2$, H_2 is presented as

$$H_2 = K\left(\frac{n}{15S_0^{1/2}h_2^{2/3}}\right)^{\alpha} \tag{21}$$

Application of Eqs. (17), (18) and (21) yields the coefficients a_2, b_2, and c_2 as

$$a_2 = 1 \tag{22a}$$

$$b_2 = 0 \tag{22b}$$

$$c_2 = \frac{-(H_0 - H_2)}{H_0(S-2)^2} \tag{22c}$$

Hence, the subsurface water profile may be expressed as

$$H(x,t) = H_0 - \frac{(H_0 - H_2)x^2}{(S-2)^2} \tag{23}$$

and the volume of water stored in subsurface is

$$V_{\mathrm{ia}} = \int_0^S H(x,t)\,\mathrm{d}x = SH_0 - \frac{(H_0 - H_2)S^3}{3(S-2)^2} \tag{24}$$

Substituting Eqs. (15) and (24) into Eq. (1) yields the solution of advance function for border irrigation.

$$q_0 t = h_0 S\left[1 - \frac{(h_0 - h_2)S^2}{3h_0(S-2)^2}\right] + H_0 S\left[1 - \frac{(H_0 - H_2)S^2}{3H_0(S-2)^2}\right] \tag{25}$$

Above is development of the model for water advance phase by Yu and Singh. With the water advance distance increases, one may assume $S = S - 2$ and Eq. (25) may be approximately and simply expressed as

$$q_0 t = \tfrac{1}{3}S[2(h_0 + H_0) + h_2 + H_2] \tag{26}$$

Eq. (26) is an implicit function of n for a given distance S and an advance time t and can be easily solved by iteration using the following equation:

$$n = \frac{60S_0^{1/2}}{q_0}\left[\frac{3q_0 t}{2S} - H_0 - \frac{1}{2}(h_2 + H_2)\right]^{5/3} \tag{27}$$

For K and α in Kostiakov infiltration equation (Eq. (7)), according to the law of the volume balance, Wang (1994) found that they could be computed by using data of water advance and recession phase of two irrigation events in homogeneous field as

$$1000q_0 t = \int_0^{L_{\mathrm{m}}} K[t_{\mathrm{i}}(x)]^{\alpha}\,\mathrm{d}x, \quad L_{\mathrm{m}} \le L \tag{28}$$

$$t_{\mathrm{i}}(x) = t_{\mathrm{r}}(x) - t_{\mathrm{a}}(x) \tag{29}$$

where $t_{\mathrm{a}}(x)$ is water advance time; $t_{\mathrm{r}}(x)$ the water recession time; and $t_{\mathrm{i}}(x)$ the infiltration time; and L_{m} the total advance distance. Two times of irrigation get two different $t_{\mathrm{i}}(x)$ and two equations like Eq. (28). Then joining the two equations, K and α can be solved. The $t_{\mathrm{i}}(x)$ can be suited by parabolic form in which correlation coefficient reaches over 0.99.

Table 1
Soil physical properties in the experiment area

Thickness of soil layer (cm)	Bulk density (g/cm^3)	Porosity (%)	Field capacity (mm)	Available water capacity (mm)	Physical clay (%)
0–5	1.15	57.2			
5–18	1.28	52.4	146.8 (0–44 cm)	79.7 (0–44 cm)	55–60 (0–44 cm)
18–28	1.30	52.0			
28–44	1.39	48.5			

The conventional method often has been used to measure K and α by choosing several points randomly on the border and conducting infiltration experiments there; then K and α may be obtained fitting the Kostiakov infiltration equation. This has the weakness of being time consuming and inaccurate for spatially variable soil. The above method of calculating the infiltration parameters overcomes these weaknesses as it depends only on data of the water advance and recession phases on a whole border, thus, the parameters obtained reflect the average condition on the whole border, instead of several isolated points. The advantage of the method for estimating n is that it requires only the observation of the water advance and recession times, instead of the surface water depths.

2.2. Experimental design and data collection

Field experiments were conducted in cinnamon soil region in Kazuo town, western Liaoning province, China. Borders under different situations were selected, including bare soil after plowing and the same lands with crop. For measuring water advance and recession time, stakes were set at an interval of 10 m on borders. Water inflow, etc. were recorded for several irrigation events. The soil physical properties in the experiment area are listed in Table 1.

3. Results and discussion

Inputting required parameters into Eq. (27), different Manning's roughness coefficient n under different conditions can be calculated on computer.

3.1. The n-values of lands after plowing without crop

Manning's roughness coefficients for field after fall plowing without crop, calculated with Eq. (27), are shown in Table 2. Results show that bare soils have smaller n-values, ranging from 0.02 to 0.04 and average 0.03.

3.2. The n-values of the same lands with spring wheat

When spring wheat grows up, Manning's roughness is influenced not only by the soil surface but also by the crop, and n-values shown in Table 3, ranging from 0.03 to 0.05 and average 0.039 are higher than that of lands without crop obviously.

Table 2
Manning's roughness coefficients for lands after fall plowing without crop

Border no.	Inflow rate q_0 (m^3/m/min)	Bed slope S_0 (‰)	Infiltration factor		Distance S (m)	Time t (min)	n-value
			K (cm/min$^{\alpha}$)	α			
1	0.287	1.1	1.298	0.682	60	26	0.0371
2	0.229	2.3	1.322	0.671	60	27	0.0207
3	0.238	1.5	1.697	0.665	60	29	0.0297
4	0.234	2.1	1.371	0.695	60	32	0.0203
5	0.257	3.1	1.533	0.643	60	27	0.0352
6	0.234	3.2	1.353	0.644	60	27	0.0383
7	0.243	2.2	1.425	0.657	60	28	0.0282
8	0.233	2.7	1.432	0.645	60	30	0.0328
Average							0.0303
S.D.							0.0026

3.3. The n-values under the condition of intermittent infiltration

Three cycles of intermittent irrigation (cycle inflow time $t_{on} = 20$ min, intermittent time $t_{off} = 40$ min) were carried out on a border. Results listed in Table 4 show that the highest n-value is encountered during the first irrigation cycle. These n-values became smaller during the second irrigation cycle, and again smaller during the third one. This is because the watering led the loose topsoil wet and clods dissolved, at the same time, the interval between each of two irrigation events made the topsoil structure much denser and smoother than normal.

Table 3
Manning's roughness coefficients for lands with spring wheat

Border no.	Inflow rate q_0 (m^3/m/min)	Bed slope S_0 (‰)	Infiltration factor		Distance S (m)	Time t (min)	n-value
			K (cm/min$^{\alpha}$)	α			
1	0.287	1.1	1.298	0.682	60	28	0.0450
2	0.229	2.3	1.322	0.671	60	30	0.0301
3	0.238	1.5	1.697	0.665	60	31	0.0507
4	0.234	2.1	1.371	0.695	60	34	0.0257
5	0.257	3.1	1.533	0.643	60	28	0.0398
6	0.234	3.2	1.353	0.644	60	28	0.0430
7	0.243	2.2	1.425	0.657	60	29	0.0318
8	0.233	2.7	1.432	0.645	60	32	0.0458
Average							0.0390
S.D.							0.0033

Table 4
Manning's roughness coefficients for three irrigation cycles

Cycle	Inflow rate q_0 (m^3/m/min)	Bed slope S_0 (‰)	Infiltration factor		Distance S (m)	Time t (min)	The n-value
			K (cm/min$^{\alpha}$)	α			
1			1.533	0.645		20	0.027
2	0.287	4.6	0.494	0.645	60	6	0.018
3			0.494	0.645		5	0.012

3.4. The n-value for border irrigation in loess region

Manning's roughness n calculated by Eq. (27) and shown in Table 5 was compared by data (Fei, 1994) from loess irrigation region in Jinghui, Shannxi province. The result shows that the calculated n-value by the proposed method is identical to that calculated in

Table 5
Calculated n-value in loess region

Inflow rate q_0 (m^3/m/min)	Bed slope S_0 (‰)	Infiltration factor		Distance S (m)	Time t (min)	Calculated n-value	The n-value (Fei, 1994)
		K (cm/min$^{\alpha}$)	α				
0.48	3.4	0.601	0.851	160	54.7	0.042	0.042

Table 6
Average relative and absolute deviation for Manning's n-values by two different methods

Field condition	Border no.	The n-value calculated by proposed method	The n-value calculated by Saint–Venant equation	Average relative error (%)	Average absolute error
Lands without crop	1	0.0371	0.0373	0.54	0.0002
	2	0.0207	0.0204	1.47	0.0003
	3	0.0297	0.0298	0.34	0.0004
	4	0.0203	0.0203	0.00	0.0000
	5	0.0352	0.0350	0.57	0.0002
	6	0.0383	0.0382	0.26	0.0001
	7	0.0282	0.0280	0.71	0.0002
	8	0.0328	0.0331	0.91	0.0003
Lands with spring wheat	1	0.0450	0.0443	1.58	0.0018
	2	0.0301	0.0303	0.66	0.0002
	3	0.0507	0.0500	1.40	0.0007
	4	0.0257	0.0258	0.39	0.0004
	5	0.0398	0.0395	0.76	0.0003
	6	0.0430	0.0420	2.38	0.0014
	7	0.0318	0.0320	0.62	0.0002
	8	0.0458	0.0456	0.44	0.0002
Average				0.81	0.0003

Fei's Experiments Report, in which Zero Inertia Model developed by Strelkoff was applied to search some sort of optimization for n.

For different situations and various soil types, Manning's roughness coefficients n range from 0.02 to 0.05, and agree with the conclusion drawn by some scientists that n-value for border irrigation may range 0.02 to 0.4. Chow (1959) reported n-values of 0.02 as minimum, 0.03 as normal, and 0.04 as maximum for cultivated areas with no crop. Furthermore, the n-value calculated in Table 5 equals to the given value.

Meanwhile, for Tables 2 and 3, a comparison of n-value calculated by the proposed method was made with that by Saint–Venant equation (The Full Hydrodynamic Model), in which also n-values are searched for optimization. The average relative error (ARE) and average absolute error (AAE) for the 16 data sets, shown in Table 6, were only 0.81% and 0.0003, respectively. Therefore, reasonable results can be obtained by using the proposed method.

The YSM is accurate for short border (<100 m) (Yu and Singh, 1989), and as the border length increases, the model's prediction error also increases. Manning's n-value calculated by the above method has the same weakness. However, a sensitivity analysis of this parameter showed that n-value changed within 0.001 when S changed from 30 to 100 m. It may, therefore, be concluded that the present method is suitable for calculating n-values in the field for border irrigation. Although the accuracy of results by the present technique depends on the accuracy of input data, particularly the water recession time, which determines the infiltration parameters K and α, this method is still more convenient and accurate than that by measuring the water depths during water flow.

4. Conclusions

In this paper, YSM is simplified to calculate Manning's roughness. The n-values can be computed by using data of water advance and recession time and flow distance, etc. instead of surface water depth. The accuracy of n-value may be improved as well.

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